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Assignment Continous_Fourier due 01/19/2022 at 02:11pm EET

Problem 1. **(1 point)** METUNCC/Applied_Math/fourier/R_coeffs.pg

Suppose that $f(t)$ is periodic with period $[-\pi, \pi)$ and has the following **real** Fourier coefficients:

$a_0 = -2, \quad a_1 = -1, \quad a_2 = -2, \quad a_3 = 4, \quad \ldots$

$b_1 = 2, \quad b_2 = -3, \quad b_3 = -3, \quad \ldots$

(A) Write the beginning of the real Fourier series of $f(t)$ (through frequency 3):

$f(t) =$ ________________________________

(B) Give the real Fourier coefficients for the following functions:

(i) The derivative $f'(t)$

$a_0 =$ ___, $a_1 =$ ___, $a_2 =$ ___, $a_3 =$ ___, ...

$b_1 =$ ___, $b_2 =$ ___, $b_3 =$ ___, ...

(ii) The function $f(t)+1$

$a_0 =$ ___, $a_1 =$ ___, $a_2 =$ ___, $a_3 =$ ___, ...

$b_1 =$ ___, $b_2 =$ ___, $b_3 =$ ___, ...

(iii) The antiderivative of $(f(t)+1)$ (with $C = 0$)

$a_0 =$ ___, $a_1 =$ ___, $a_2 =$ ___, $a_3 =$ ___, ...

$b_1 =$ ___, $b_2 =$ ___, $b_3 =$ ___, ...

(iv) The function $f(t)+2\sin(t)+2\cos(2t)$

$a_0 =$ ___, $a_1 =$ ___, $a_2 =$ ___, $a_3 =$ ___, ...

$b_1 =$ ___, $b_2 =$ ___, $b_3 =$ ___, ...

(iv) The function $f(2t)$

$a_0 =$ ___, $a_1 =$ ___, $a_2 =$ ___, $a_3 =$ ___, ...

$b_1 =$ ___, $b_2 =$ ___, $b_3 =$ ___, ...

Problem 2. **(1 point)** `METUNCC/Applied_Math/fourier/C_coeffs.pg`

Suppose that $f(t)$ is periodic with period $[-\pi, \pi)$ and has the following **complex** Fourier coefficients:
$\ldots \quad c_0 = 2, \quad c_1 = 3-4i, \quad c_2 = -4i, \quad c_3 = -1+2i, \quad \ldots$

(A) Compute the following complex Fourier coefficients.
$c_{-3} =$ ___, $c_{-2} =$ ___, $c_{-1} =$ ___

(B) Compute the real Fourier coefficients. (Remember that $e^{ikt} = \cos(kt) + i\sin(kt)$.)
$a_0 =$ ___, $a_1 =$ ___, $a_2 =$ ___, $a_3 =$ ___, ...
$b_1 =$ ___, $b_2 =$ ___, $b_3 =$ ___, ...

(C) Compute the complex Fourier coefficients of the following.

(i) The derivative $f'(t)$.
$c_0 =$ ___, $c_1 =$ ___, $c_2 =$ ___, $c_3 =$ ___

(ii) The shifted function $f\left(t + \frac{\pi}{6}\right)$
$c_0 =$ __________, $c_1 =$ __________,
$c_2 =$ __________, $c_3 =$ __________

(iii) The function $f(3t)$.
$c_0 =$ ___, $c_1 =$ ___, $c_2 =$ ___, $c_3 =$ ___